A More Realistic AC-QSS Cascading Failure Model with Decentralized UVLS and Centralized RAS

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***Abstract*—Currently, the divergence problem of AC-Quasi-Steady-State (QSS) cascading failure models is addressed by uniform load shedding (ULS) that trips load in same fraction at all buses till convergence is achieved. However, in reality there exist pre-designed undervoltage load shedding (UVLS) relays that shed load in the subset of buses where voltage descends bellow a threshold, thereby preventing voltage collapse. This might not be captured by ULS, which in turn leads the cascade propagation to a different path. We attack the problem by proposing a new AC-QSS model, which combines voltage magnitude and Q-V sensitivity to predict buses that have undergone UVLS tripping in reality. Moreover, we introduce a centralized AC optimal preventive control approach to alleviate cascade propagation. We use a simple AC-QSS model with ULS as benchmark to contrast results of UVLS model and further demonstrate the effectiveness of preventive control on IEEE 118-bus system and a 2383-bus Polish network.**

***Index Terms*—AC-QSS model, Cascading failure, Undervoltage load shedding, UVLS, Optimal preventive control, Voltage collapse.**

I. Introduction

UNDERVOLTAGE load shedding (UVLS) is one of the protective schemes used to recover voltage profile to an acceptable level, thereby preventing propagation of cascading failure in a large scale and leading to a voltage collapse [1]–[4]. Designing UVLS strategies require coordination among protection engineers and system planners who determine the amount and time delay required for load shedding based on metrics such as proximity to nose point in P-V curves [5]. In this paper, we consider a decentralized scheme with UVLS relays at buses in the network. The UVLS relays will trip loads in a pre-designed fraction at buses where voltage magnitude have dropped below a pre-determined threshold and remain there for a certain time. Such UVLS schemes are extensively investigated by Western Electricity Coordinating Council (WECC) [4]. We underline the fact that the cascading failure models that do not include UVLS action deviate from the ground truth. In this regard, we highlight that the pre-existing UVLS scheme is not considered in most of the current AC-QSS models [6]–[15]. In the case of divergence, these models typically use uniform load shedding (ULS) that trips loads at all buses (irrespective of voltage) in the same proportions to achieve a converged power flow (PF) [7], [10], [12], [13]. We emphasize that, although the converged PF solution might satisfy the numerical convergence for PF problem, it does little to indicate a realistic load shedding pattern that happens following cascading failure by UVLS relays. The proposed model in [14] uses continuation power flow (CPF) [5] to deal with the divergence problem. However, based on authors' experience, CPF might lead to different $P-V$ curves depending on the starting points, which was never discussed.

In this work, we also take into account a centralized remedial action scheme (RAS) for cascade prevention that applies generation rescheduling and load shedding action. For cascade mitigation, some of the existing literature have included DC-QSS-based preventive control, which has been evaluated on an AC-QSS model [8], [9]. To the best of the authors' knowledge, an AC model for optimal preventive control of cascading failure has not been proposed before.

The contribution of this paper is twofold. First, we propose an AC-QSS cascading failure model to include the pre-existing decentralized UVLS relays. To that end, we predict the buses where UVLS shedding takes place during cascade propagation by using an index that combines the voltage magnitudes and sensitivity/weakness of buses. Second, for cascade mitigation, we propose a novel RAS scheme based on AC-QSS-based optimal preventive control. We investigate the proposed models (UVLS with/without optimal preventive control) together with a simple AC-QSS model with ULS as benchmark on IEEE 118-bus system and 2383-bus Polish network.

II. AC-QSS Model: Areas of Improvement

In this section, first we take a glance into state-of-art AC-QSS models [7], [10], [11], [13] to analyze divergence problem and some other existing issues including preventive control, see [15] for further details. Typically, cascading failure is triggered by some initial failures or outages that might break the network into different islands/sub-networks, which are studied separately. If PF diverges in an island, the current models start shedding load uniformly in all buses until convergence is achieved. Cascade propagates in an island by tripping overloaded lines in each tier and stops when there is no overloaded line, or there is no more load for shedding, indicating complete blackout. In this paper, as a benchmark,

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we use a typical AC-QSS cascade model with ULS including above-mentioned steps.

In a real world cascading failure, the existing UVLS relays in the network shed a pre-designed fraction of load in the buses where corresponding voltage magnitudes fall below a voltage threshold and remain there for a pre-specified time, normally 3-5 s [1]–[4]. This voltage dependent load shedding by UVLS relays, which is completely different from ULS, aims to prevent voltage collapse, reflecting ground truth. However, including the UVLS scheme in AC-QSS model is challenging due to the divergence issue that might happen following cascading failure. As mentioned earlier, the other mechanism of preventing cascading failure is centralized RAS. The existing literature on this topic considered DC-QSS-based preventive control and tested it on AC-QSS model [8], [9].

Thus the AC-QSS models in the existing literature have two important areas that require improvement. We note that it is not feasible to capture the exact ground truth by AC-QSS model due to its inherent limitations compared to the detailed dynamic counterpart. Nevertheless, improving the AC-QSS model to better reflect the ground truth is an open problem, which is the focus of this paper.

III. Proposed AC-QSS Model

In this section, we present the proposed AC-QSS model to include pre-existing UVLS and centralized RAS-based optimal preventive control (OPC) during cascade propagation, which is built upon our preliminary work on this topic [16]. Figure 1 depicts the flowchart of the model, which can be divided into three interconnected sections – SC_{Body}, SC_{Conv}, and SC_{Div}. SC_{Body} or body-section of the model includes some typical functions of AC-QSS cascade model such as application of initial node outages, island formation, and overloaded line tripping. SC_{Conv} and SC_{Div} represent the sections that deal with islands with converged and diverged PFs, respectively, the behavior of UVLS relays and centralized RAS. Figure 2 shows the time sequence of UVLS and OPC for the proposed model. We study our QSS model in snapshots every t_s seconds and consider potential UVLS actions, while OPC is applied every T_{OPC} seconds, which is an integral multiple of t_s. The value of t_s is determined by the delay in UVLS tripping while T_{OPC} depends on the delays of state estimation and actuation.

A. Inclusion of UVLS action

When the voltage magnitude of a load bus i drops below a pre-selected voltage threshold v_{th} for a certain period of time t_s, the UVLS relay sheds a fraction of load $1-\gamma$ as follows:

$$P_{i,load} + jQ_{i,load} = \gamma^{N^i_{shed}}(P^{pre}_{i,load} + jQ^{pre}_{i,load}),\ \forall i \\ N^i_{shed} \in [0, 1, ..., N^{max}_{shed}] \tag{1}$$

where $P_{i,load}$ ($P^{pre}_{i,load}$) and $Q_{i,load}$ ($Q^{pre}_{i,load}$) show post-shedding (pre-disturbance) active and reactive power loads at bus i, respectively. Moreover, the integers N^i_{shed} and N^{max}_{shed} are number of trippings at bus i and maximum allowable load

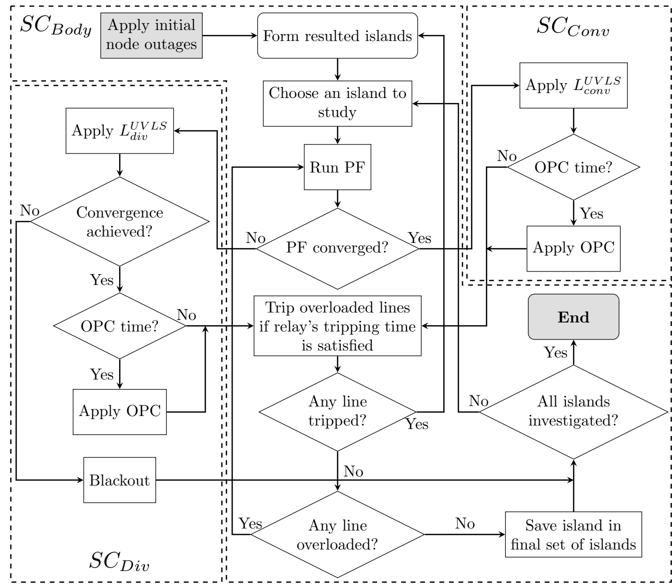

Fig. 1. Flowchart of the proposed model.

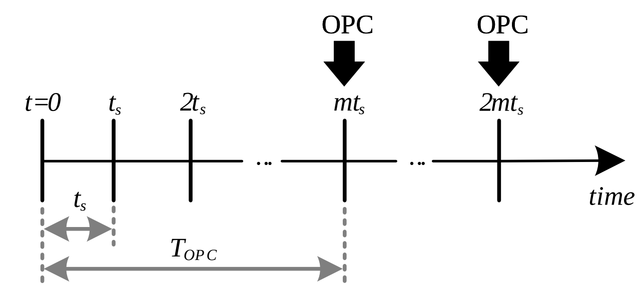

Fig. 2. Time sequence of UVLS and OPC in the proposed model.

Algorithm 1: Converged Layer of UVLS (L^{UVLS}_{conv})

1 If there exists any bus i such that $v_i \leq v_{th}$ and $N^i_{shed} \leq N^{max}_{shed}$, go to 2. Otherwise, go to 4.
2 Identify candidate buses for load tripping.
3 Trip load once in candidate buses. Go to 1.
4 Either no more tripping is required or possible. Hence, L^{UVLS}_{conv} is applied.

trippings. Note that v_{th}, γ, and N^{max}_{shed} are pre-designed values, and we use them as input in our model.

For islands with converged PF, first we apply converged layer of UVLS (L^{UVLS}_{conv} block) – see Fig. 1. Detailed explanations for L^{UVLS}_{conv} are provided in Algorithm 1. If the required time for OPC is reached, we conduct OPC which will be elaborated later on.

For diverged cases, SC_{Div} is proposed that starts with diverged layer of UVLS (L^{UVLS}_{div} block) presented in Algorithm 2. After application of L^{UVLS}_{div}, we check that the convergence is achieved or not. If convergence is not achieved, the blackout is recorded for the island. Otherwise, we perform OPC at the desired instants, see Fig. 2.

First, we identify the candidate buses for load shedding in

Algorithm 2: Diverged Layer of UVLS (L_{div}^{UVLS})

1 Save diverged network as $SubNet_{init}^{div}$. This shows the intact diverged island.
2 Run ULS till convergence is achieved. If convergence is reached, go to 3. Otherwise, go to 11.
3 Use PF results of converged island after applying ULS to identify the candidate buses for load tripping.
4 If there exists at least one candidate bus i such that $N_{shed}^{i} \leq N_{shed}^{max}$, go to 5. Otherwise, go to 11.
5 Trip load once in the candidate buses in $SubNet_{init}^{div}$.
6 Run PF.
7 If PF diverged, go to 4. Otherwise, go to 8.
8 If there is any bus i such that $v_i \leq v_{th}$ and $N_{shed}^{i} \leq N_{shed}^{max}$, go to 9. Otherwise, go to 11.
9 Identify candidate buses for load tripping.
10 Trip load once in candidate buses. Go to 8.
11 Either no more tripping is required or possible. Hence, L_{div}^{UVLS} is applied.

UVLS. To this end, we combine the weakness index with voltage magnitude of buses to calculate a new metric. It is worthwhile to mention that a converged solution of PF is required to calculate the metric. In this regard, we either directly use the solution of PF for converged cases, or leverage the converged solution of ULS for diverged cases – if there exists one.

We utilize the $Q-V$ sensitivity of buses [17] in a network to calculate weakness index:

$$\frac{\partial v_i}{\partial Q_i} = \sum_{k} \frac{\xi_{ki}\eta_{ik}}{\lambda_k} = \sum_{k} \frac{\mathcal{Z}_{ik}}{\lambda_k}, \ \forall i \tag{2}$$

where, $\mathcal{Z}_{ik} = \xi_{ki}\eta_{ik}$. ξ_{ki}, and η_{ki} are the i^{th} elements of the right and left eigenvector of the reduced Jacobian matrix of the system for mode k, i indicates the bus number, λ_k is the eigenvalue corresponding to k^{th} mode, and $\mathcal{Z}_{ki}$ is the weakness index of bus i for mode k. The smaller eigenvalues in a system are associated with the weakest modes of system. Here, we use the smallest eigenvalue [18]. A higher value for $\mathcal{Z}_i$, shows higher sensitivity for bus i.

We propose following index to identify the candidate buses:

$$\Psi_i = \frac{v_{th}-v_i}{\underset{i}{max}\,|v_{th}-v_i|} + \frac{\mathcal{Z}_i}{\underset{i}{max}(\mathcal{Z}_i)} \tag{3}$$

We claim that higher the value of Ψ_i for bus i due to low voltage magnitude v_i and/or high weakness index $\mathcal{Z}_i$ the higher the likelihood of bus i to be a candidate bus for load shedding. To finalize the list of candidate buses, first we sort buses according to Ψ_i in the descending order. Then, we identify the last bus f in which $v_f - v_{th} - \beta \leq 0$, where β is a hyper parameter determining the conservativeness. In the sorted list, buses 1 to f form our final candidate buses for load shedding in UVLS scheme.

B. Centralized RAS: AC optimal preventive control

The proposed OPC for cascade mitigation minimizes a weighted sum of load shedding and overload of lines. Moreover, the model transforms the hard inequality constraint for line overloading to a soft constraint. The OPC model for n^{th} tier of cascade is formulated as follows:

$$\min_{\mathbf{P^n_{load}},\mathbf{P^n_{gen}},\mathbf{S^n_{over}}} -\mathbf{1^T P^n_{load}} + \lambda^{\mathbf{T}}\mathbf{S^n_{over}} \tag{4}$$

subject to:

$$|\mathbf{S^n}| \leq \mathbf{S^{max}} + \mathbf{S^n_{over}}, \quad \mathbf{S^n_{over}} \geq 0 \tag{5}$$

$$P^{n-1}_{i,load}Q^{n}_{i,load} = P^{n}_{i,load}Q^{n-1}_{i,load}, \ \forall i \tag{6}$$

$$\mathbf{P^n_{gen}} - \mathbf{P^n_{load}} - \mathbf{P^n}(\mathbf{v^n},\theta^{\mathbf{n}}) = 0 \tag{7}$$

$$\mathbf{Q^n_{gen}} - \mathbf{Q^n_{load}} - \mathbf{Q^n}(\mathbf{v^n},\theta^{\mathbf{n}}) = 0 \tag{8}$$

$$P^n_i(v^n,\theta^n) = \sum_{k=1}^{n_b} v^n_i v^n_k (G_{ik}cos\theta^n_{ik} + B_{ik}sin\theta^n_{ik}), \forall i \tag{9}$$

$$Q^n_i(v^n,\theta^n) = \sum_{k=1}^{n_b} v^n_i v^n_k (G_{ik}sin\theta^n_{ik} - B_{ik}cos\theta^n_{ik}), \forall i \tag{10}$$

$$v^{min} \leq v^n_i \leq v^{max}, \ \forall i \tag{11}$$

$$\mathbf{0} \leq \mathbf{P^n_{load}} \leq \mathbf{P^{n-1}_{load}} \leq \mathbf{P^{pre}_{load}} \tag{12}$$

$$\mathbf{P^{min}_{gen}} \leq \mathbf{P^n_{gen}} \leq \mathbf{P^{n-1}_{gen}} \leq \mathbf{P^{max}_{gen}} \tag{13}$$

$$\mathbf{Q^{min}_{gen}} \leq \mathbf{Q^n_{gen}} \leq \mathbf{Q^{max}_{gen}} \tag{14}$$

where, bold terms are vectors and $|.|$ denotes the element-wise absolute value. $\mathbf{S}$ and $\mathbf{S^{max}}$ show vector of apparent powers and maximum allowable apparent power flows of lines; $\mathbf{S_{over}}$ is a user-defined variable showing the magnitude of overload in apparent power of lines; $\mathbf{P_{load}}$ and $\mathbf{Q_{load}}$ show vectors of active and reactive loads; $\mathbf{P_{gen}}$ and $\mathbf{Q_{gen}}$ are vectors of active and reactive generations; P_i and Q_i denote active and reactive power injections in bus i; $\theta_{ik} = \theta_i - \theta_k$ where θ_i is voltage angle of bus i; G_{ik} and B_{ik} are conductance and susceptance of the line connecting buses i and k; $\mathbf{P^{min}_{gen}}$ and $\mathbf{P^{max}_{gen}}$ ($\mathbf{Q^{min}_{gen}}$ and $\mathbf{Q^{max}_{gen}}$) are vectors of minimum and maximum allowable active (reactive) power generations and, v^{min} and v^{max} represent minimum and maximum permitted voltage magnitudes.

Equation (5) is included in OPC model to convert the line flow constraint into a soft constraint; (6) is to preserve a constant power factor at bus i; (7)-(10) are the active and reactive power constraints at each bus; the allowable voltage range is defined in (11); (12) indicates the OPC model is restricted to retain load at a generic tier between zero and load at previous tier of cascade, for all buses; (13) and (14) are constraints for active and reactive power generations.

The OPC model (4)-(14) tries to minimize load shedding and line overloading. The output is $[\mathbf{\hat{P}^n_{load}}, \mathbf{\hat{P}^n_{gen}}]$, which are setpoints for loads and generations. We note that OPC is applied only on the islands with converged PF. Restoration of a collapsed island is out of scope of this paper.

We remark that the proposed AC OPC model and the security-constrained optimal PF (SCOPF) [19]–[21] are fundamentally different. The SCOPF computes generation redispatch minimizing power generation cost while satisfying security constraints, including voltage and thermal limits. Moreover, SCOPF typically excludes load shedding in its formulation.

IV. Case Study

The IEEE 118-bus network and the Polish system during winter 1999 – 2000 peak condition [22] are studied here. Three models are investigated – ULS, UVLS, and UVLSPC. When divergence is observed, ULS iteratively sheds load in the blocks of 0.5% uniformly in all load buses until convergence is achieved (if possible). UVLS and UVLSPC denote proposed AC-QSS models without and with OPC, respectively. For solving OPC, Matpower [22] is used with 'fmincon' solver. For the IEEE 118-bus system, three cases with different initial node outages 1%, 5%, and 10% of total nodes are tested. Also, three cases with initial node outages 1%, 3%, and 5% are studied for the Polish system. Each case includes 500 sets of random initial node outages separately performed as Monte Carlo simulations. For UVLS and UVLSPC, $v_{th} = 0.91 \times 0.95 = 0.8645$ pu, $N_{shed}^{max} = 5$, and $\gamma = 0.75$ are assumed. For IEEE 118-bus system, $\beta = 0.02$ and for Polish network $\beta = 0.05$ is assumed. The time resolution of AC-QSS model which is time between two consecutive snapshots is assumed to be $t_c = 5$ s and $T_{OPC} = 30$ s. Box-whisker plots are used to illustrate results where small dots represent outliers, which are outsides of the whiskers, the line inside boxes shows the median value, and mean values are indicated by large dots.

A. IEEE 118-bus system

*1) **ULS vs UVLS:*** Figure 3 depicts mean voltage magnitudes for different AC-QSS models. For meaningful comparison, voltage magnitudes of all buses of network are considered – including the collapsed buses. It can be seen that the average values of mean voltage magnitudes are higher for ULS compared to the UVLS. Also, median values are almost higher for ULS. The inter-quartile range of boxes for UVLS are considerably higher compared to the ULS. Figure 4 shows that the mean load served at the end of cascade is higher for ULS with low inter-quartile range while the median values are almost identical. We note that the load served after cascade does not represent the complete picture of cascade propagation. Table I shows that the number of complete blackouts (total load served less than 1%), is much higher for UVLS than ULS.

*2) **UVLS vs UVLSPC:*** Figures 3 and 4 show that applying OPC to the UVLS model can significantly improve results of UVLS. UVLSPC results in higher mean and median values – both for voltage magnitudes and load served at the end of cascade with respect to the UVLS. Table I indicates that UVLSPC is able to decrease number of blackouts. However, the improvement diminishes as the scale of initial failures increases.

B. Polish network

*1) **ULS vs UVLS:*** Figure 5 shows boxplots of mean voltage magnitudes for Polish system. It can be seen the average of mean values are higher for ULS compared to UVLS while median values are almost identical. The inter-quartile range for UVLS noticeably increases for higher percentage of initial node outages. Figure 6 shows load served at the end of cascade for Polish system. It can be seen that mean and median values are higher for UVLS, the inter-quartile range for UVLS increases for larger number of initial node outages – whereas it is almost same for ULS. We underline the fact that the behavior of ULS and UVLS will be different in different networks. Table I provides detailed results for the Polish system. It can be seen that the number of blackouts are significantly higher for UVLS.

*2) **UVLS vs UVLSPC:*** Figures 5 and 6 indicate that OPC can considerably improve results of cascading failure. It can be observed that UVLSPC leads to higher mean and median values both for average voltage magnitudes and served load at the end of cascade. Table I exhibits that UVLSPC is capable of noticeably diminishing the number of blackouts. However,

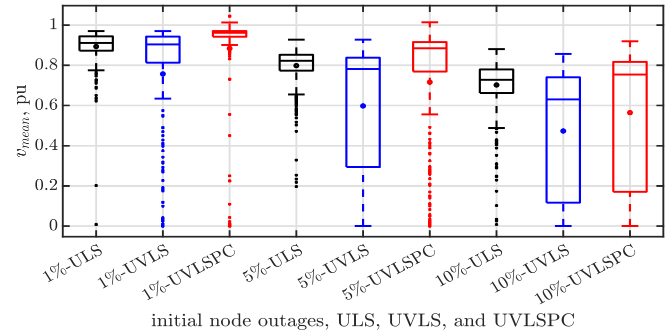

Fig. 3. Boxplots of mean voltage magnitudes at the end of cascade comparing ULS, UVLS, and UVLSPC in IEEE 118-bus system: 500 random sets are investigated for each % of initial node outages.

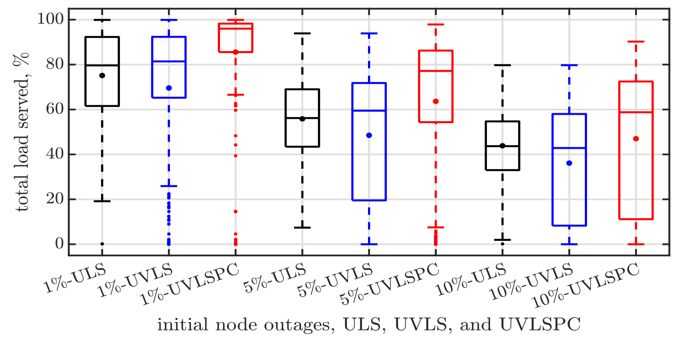

Fig. 4. Boxplots of total load served at the end of cascade comparing ULS, UVLS, and UVLSPC in IEEE 118-bus system: 500 random sets are investigated for each % of initial node outages.

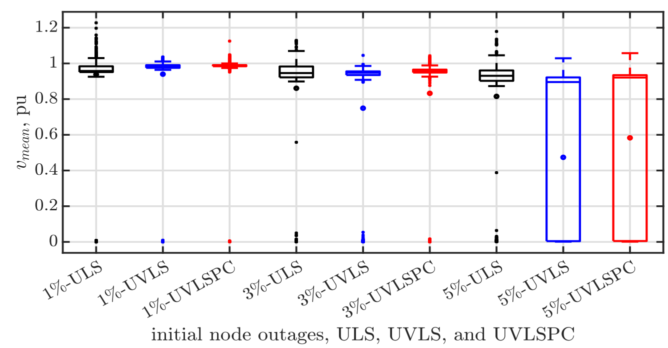

Fig. 5. Boxplots of mean voltage magnitudes at the end of cascade comparing ULS, UVLS, and UVLSPC in Polish network: 500 random sets are investigated for each % of initial node outages.

TABLE I
CASCADE PROPAGATION COMPARISON FOR ULS, UVLS, AND UVLSPC: IEEE 118-BUS SYSTEM AND POLISH NETWORK

case	percentage outage	IEEE 118-bus			percentage outage	Polish network		
		μ_I^*	μ_T^{**}	N_{BO}^{***}		μ_I	μ_T	N_{BO}
ULS		12	13	2		33	3	20
UVLS	1%	11	13	60	1%	32	2	23
UVLSPC		6	4	30		32	1	5
ULS		22	15	0		96	4	47
UVLS	5%	22	15	50	3%	96	4	101
UVLSPC		16	6	46		94	1	67
ULS		32	13	1		161	4	50
UVLS	10%	32	13	52	5%	160	4	217
UVLSPC		27	7	48		157	2	171

* Mean number of islands ** Mean number of tiers of cascade
*** Number of blackouts

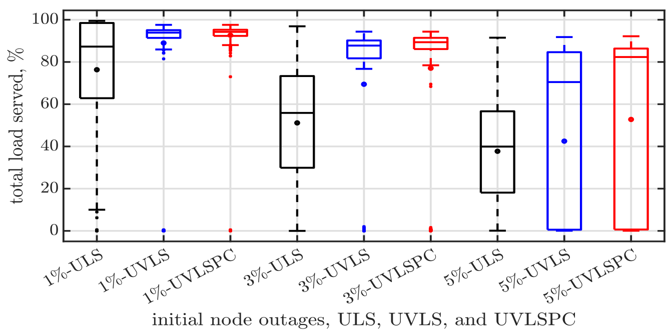

Fig. 6. Boxplots of total load served at the end of cascade comparing ULS, UVLS, and UVLSPC in Polish network: 500 random sets are investigated for each % of initial node outages.

with higher (5%) initial node outage, the effectiveness of OPC markedly deteriorates, which is consistent with the observations in the 118-bus system.

C. Cascade Propagation Path

We have performed probabilistic analysis using two metrics: distribution of number of lines out and demand loss [15], which are shown in Fig. 7. Probability of outage of each bus is assumed to be 0.0005. The metrics reveal different paths for cascade propagation in ULS and UVLS in addition to the effectiveness of control in UVLSPC with respect to UVLS.

V. CONCLUSION

This paper proposed a new model for inclusion of pre-existing UVLS relays in the AC-QSS model for cascading failure. In addition, a novel AC optimal preventive control for cascade mitigation was formulated and included in the proposed cascade model. Extensive Monte Carlo studies were performed on IEEE 118-bus system and 2383-bus Polish network. Contrasting results of UVLS against ULS model used in the literature discloses that ULS leads the cascading failure into a completely different path compared to UVLS. Moreover, results indicate the effectiveness of proposed optimal preventive control model to limit propagation of cascading failure.

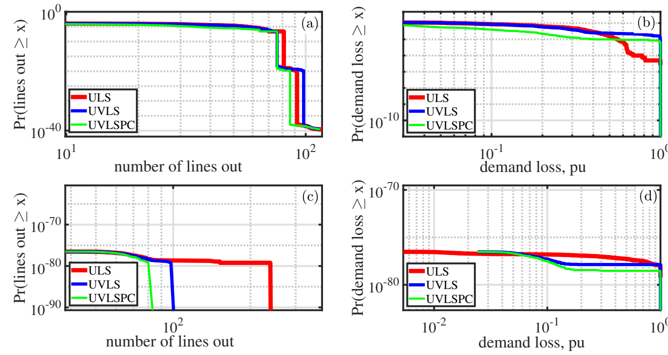

Fig. 7. Distribution of number of lines out: (a) IEEE 118-bus system, (c) Polish network. Distribution of demand loss: (b) IEEE 118-bus system, (d) Polish network.

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